# Erratum: Equation of state for a partially ionized gas. II [Phys. Rev. E 68, 056112 (2003)]

George A. Baker, Jr.

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There was a miscoding in this paper which has been corrected in this Erratum. I update those numerical results reported previously that have changed outside graphical accuracy.

First, Fig. 4, which shows the deviations from the low temperature limit, is slightly changed, but the conclusion that they are proportional to $1/Ty^{12}$ continues to hold. The revised figure is given here.

Table I shows the critical point values, which I have determined for the spherical cellular model. The temperature is in electron volts, the density is in grams per cubic centimeter, and the pressure is in megabars. $\mathcal{A}$ is the gram atomic weight. The deBroglie density $\zeta$ and $y$ are defined as

$$\zeta = \frac{ZN}{2\Omega}\left(\frac{h^2}{2\pi mkT}\right)^{3/2}, \quad y^2 = \frac{Ze^2}{r_b kT}, \quad \frac{4\pi}{3} r_b^3 = \frac{\Omega}{N},$$

where $N$ is the number of ions, $\Omega$ is the volume, $T$ is the temperature, $e$ is the electron charge, $m$ is the electron mass, and $k$ is the Boltzmann constant. The elements reported are hydrogen $Z=1$, lithium $Z=3$, nitrogen $Z=7$, sodium $Z=11$, potassium $Z=19$, nickel $Z=28$, rubidium $Z=37$, palladium $Z=46$, cesium $Z=55$, and erbium $Z=68$. The ratio $p_c\Omega_c/ZNkT_c$ relates to the law of corresponding states and is not completely constant. The ratio $Z\rho/\mathcal{A}$ is the electron density. In the Thomas-Fermi-Dirac model [1,2] there is also a critical point. It is at about 0.42 eV, independent of $Z$ [3].

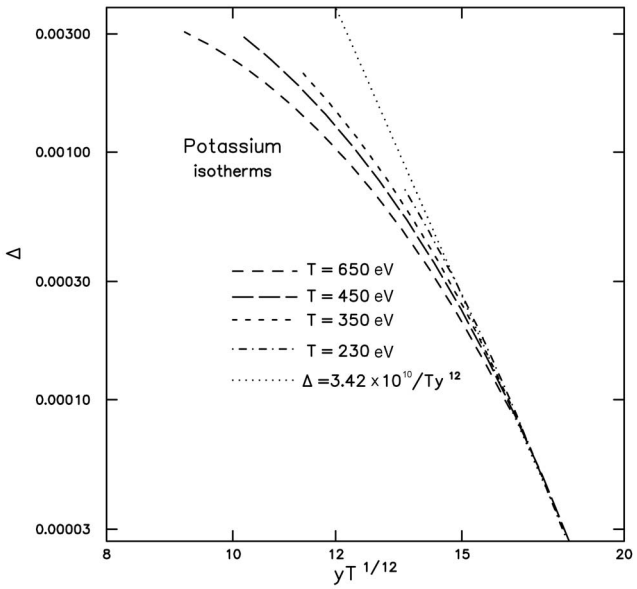


FIG. 4. The deviation $\Delta = 1 - [p_{\text{electron}}(\rho,T)p_{\text{ideal}}(\rho,1\text{ eV})]/[p_{\text{electron}}(\rho,1\text{ eV})p_{\text{ideal}}(\rho,T)]$ from the zero temperature limit (approximated here by the 1 eV curve) of the pressure for several potassium ($Z=19$) isotherms at high density.

 

TABLE I. Critical point values. $T_c$ is in units of eV, $\rho_c$ in g/cm$^3$, $\mathcal{A}$ is the gram atomic weight, $p_c$ in Mbars, and $Z\rho_c/\mathcal{A}$ in units of $0.6022\times10^{24}$/cm$^3$.

| $Z$ | 1, H | 3, Li | 7, N | 11, Na | 19, K | 28, Ni | 37, Rb | 46, Pd | 55, Cs | 68, Er |
|---|---|---|---|---|---|---|---|---|---|---|
| $T_c$ | 1.854 | 18.35 | 62.71 | 111.4 | 215.2 | 376.6 | 592.1 | 670.2 | 1064 | 1644 |
| $\rho_c$ | 0.0986 | 4.159 | 54.87 | 158.2 | 617.0 | $2.030\times10^3$ | $4.61\times10^3$ | $8.52\times10^3$ | $1.407\times10^4$ | $2.684\times10^4$ |
| $\mathcal{A}$ | 1.00797 | 6.94 | 14.007 | 22.977 | 39.0963 | 56.71 | 85.4678 | 106.4 | 132.91 | 167.26 |
| $\frac{p_c\Omega_c}{ZNkT_c}$ | 0.486 | 0.419 | 0.543 | 0.718 | 1.292 | 1.451 | 1.906 | 2.627 | 2.489 | 2.468 |
| $\frac{p_c}{}$ | $8.96\times10^{-2}$ | 1.434 | 735 | $6.06\times10^3$ | $8.34\times10^4$ | $5.48\times10^5$ | $2.253\times10^6$ | $6.49\times10^6$ | $1.543\times10^7$ | $4.43\times10^7$ |
| $y_c$ | 2.21 | 1.629 | 1.763 | 1.919 | 2.084 | 2.192 | 2.152 | 2.41 | 2.189 | 2.099 |
| $\zeta_c$ | 3.87 | 2.235 | 4.14 | 6.43 | 9.48 | 13.69 | 13.83 | 21.20 | 16.73 | 16.34 |
| $Z\rho_c/\mathcal{A}$ | $9.79\times10^{-2}$ | 1.798 | 27.4 | 75.7 | 300 | $1.002\times10^3$ | $1.997\times10^3$ | $3.69\times10^3$ | $5.82\times10^3$ | $1.091\times10^4$ |

The spherical cellular model's critical temperature is roughly given by $T_c=2.61Z^{1.5}$. Its critical density is roughly given by $\rho_c=0.245Z^{2.75}$; its critical pressure is given by $p_c=0.0313Z^5$; its critical value of $y$ is roughly 2; and its critical value of the electron density by $Z\rho_c/\mathcal{A}=0.1Z^{2.75}$, whose representation is a bit more accurate than the one for $\rho_c$.

The updated results reported in this Erratum affect the numerical results reported in the original paper, as well as in [4] and [5]. In the original paper, the table is replaced by the current Table I. Figures 6, 8, and 18 in that paper should now be considered as illustrative only and not quantitative, Fig. 7 is no longer relevant, and Figs. 9–13 are replaced by the tabular results on the critical properties reported above. The updated versions of Figs. 14–16 of that paper showing the phase

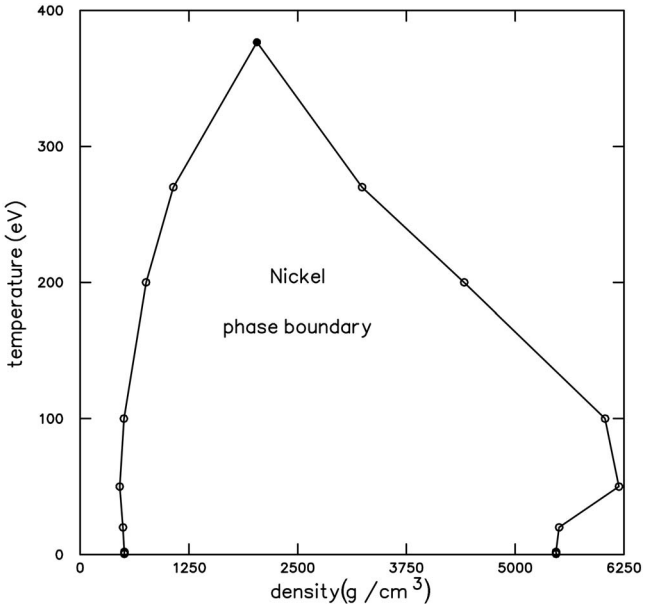


FIG. 14. The phase boundary for nickel as computed for the spherical cellular model. The open circles are the data points and the large dot is the critical point.

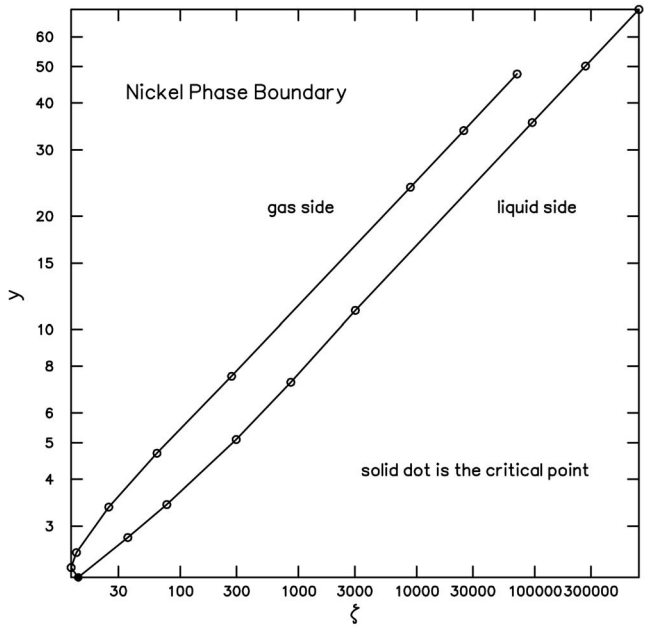


FIG. 15. The phase boundary for nickel as computed for the spherical cellular model. The open circles are the data points and the large dot is the critical point.

boundaries for nickel ($Z=28$) and other elements are given here. Note is taken that triple points are no longer present. The asymptotic behavior for large $\zeta$ corresponds to the phase boundary terminating a definite density as $T\rightarrow 0$. For $T=0$ (approximated here by 2 eV) the liquid-side density is roughly $\rho_{\text{liquid}}=0.284Z^3$ and the gas-side density is roughly $\rho_{\text{gas}}=0.012\ 85Z^{3.2}$. Hydrogen is an exception where the gas-side phase boundary behaves something like $T\propto\rho^3$. Its liquid side goes to a fixed density. Figure 17 is replaced by the figure given here.

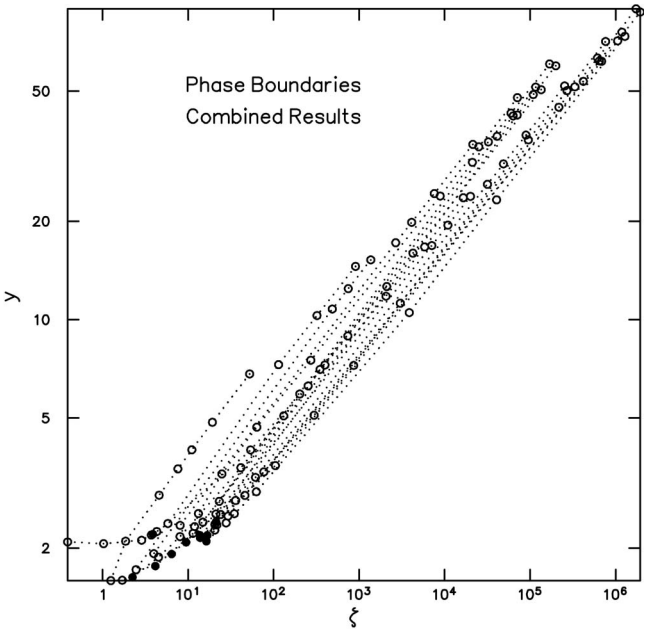


FIG. 16. The phase boundaries of the spherical cellular model for 10 different elements. The open circles are the data points and the large dots are the critical points. The points for each element are joined by dotted lines. Except for hydrogen, there seems to be no two phase regions for $\zeta<1$.

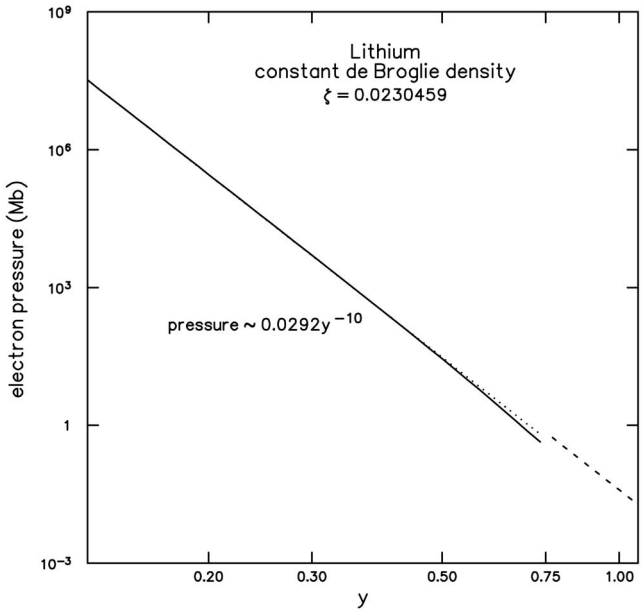


FIG. 17. The pressure of the spherical cellular model of lithium ($Z=3$) at constant de Broglie density $\zeta=0.023\,045\,9$. The solid line is the spherical model result and the dotted line (mostly obscured by the solid line) is the formula shown. The short dashed line is a continuation of the expected large $y$ limiting behavior.

One of the more dramatic changes is the replacement of Figs. 19 and 20 by the figures given here. The large difference seen previously between the internal energy and the "physically reasonable energy" is now very small. The difference between the two energies is due to the effects of the effective mass, which, in these two sample cases, is rather small. The numerical results in the original paper not explicitly mentioned here do not appear to deviate beyond graphical accuracy.

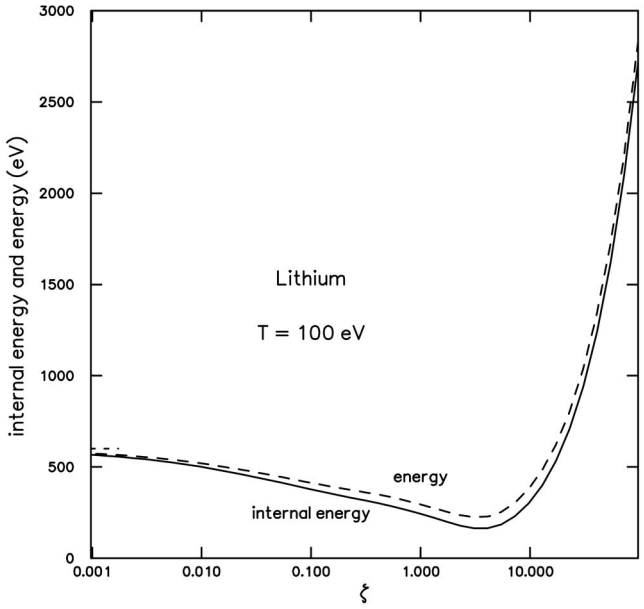


FIG. 19. The internal energy and the "physically reasonable energy" labeled energy for lithium. These are computed for the spherical cellular model.

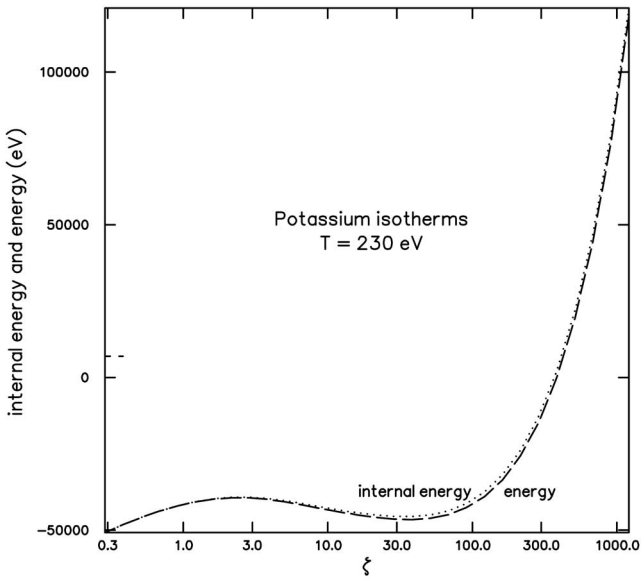


FIG. 20. The internal energy and the "physically reasonable energy" labeled energy for potassium. These are computed for the spherical cellular model.

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